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Technical Note

Surface temperature of a flat plate of finite thickness under conjugate laminar forced convection heat transfer condition

Kikuji Chida

Department of Mechanical Engineering, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama 221-8686, Japan

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1. Introduction

One of the classical conjugate problems is a heat transfer between a laminar forced convection and a flat plate of finite thickness, the bottom of which is kept at a constant temperature. There have been several reports on this problem, that is, Luikov [1], Payvar [2], Karvinen [3], Pozzi and Lupo [4], Pop and Ingham [5], Treviño et al. [6] and Vynnycky et al. [7]. In Refs. [1–5], heat conduction in the flat plate is assumed to be one-dimensional. Recent studies [6,7] take account of axial conduction in the flat plate.

Examination of the surface temperatures given by the above-mentioned studies reveals that there are two solution families on the surface temperature for the case of one-dimensional heat conduction in the flat plate: one by Refs. [1–3], the other by Refs. [4,5]. The difference of the two solution families will be shown later. The effect of the two-dimensional heat conduction on the surface temperature of the flat plate, which appears near the leading edge, is not yet made so clear.

This study tries to get a clear view of this classical conjugate problem through examination of the behavior of the surface temperature with distance from the leading edge. After deriving dimensionless groups which determine the surface temperature of the flat plate, the surface temperatures of the flat plate are calculated numerically, then compared with the solutions of the available studies.

2. Governing equations

A schematic diagram of the present problem is shown in Fig. 1. A uniform flow, whose velocity and temperature are u_{in} and T_{in}, respectively, flows over a flat plate of finite thickness e. The laminar boundary layer approximation is assumed to be valid in the fluid. The bottom of the plate (at $z=e$, the z-coordinate is directed downward from the flat plate surface) is maintained at a constant temperature T_b. The forward surface of the plate (at $x = 0$) is assumed to be adiabatic. The surface temperature of the plate (at $y=z=0$), T_w, is studied under a conjugate thermal condition.

For the fluid flow, the well-known Blasius's equation and the boundary conditions (f is the dimensionless stream function and $\eta = y(u_{in}/\nu_f x)^{1/2}$, see Nomenclature)

$$\frac{d^3 f}{d\eta^3} + \frac{1}{2} f \frac{d^2 f}{d\eta^2} = 0$$

$$\eta = 0:\quad f = 0,\quad df/d\eta = 0;\quad \eta = \infty:\quad df/d\eta = 1 \tag{1}$$

are used [8].

The energy conservation equations for the fluid and the flat plate in dimensional form are given by

$$u\frac{\partial T_f}{\partial x} + v\frac{\partial T_f}{\partial y} = a_f \frac{\partial^2 T_f}{\partial y^2} \tag{2}$$

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial z^2} = 0 \tag{3}$$

where T_f, T_s are the temperatures in the fluid and the flat plate, respectively, a_f is the thermal diffusivity of the fluid, and u, v are velocity components. The boundary conditions are set as follows

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Nomenclature

a_f	thermal diffusivity of the fluid
Br_x	Brun number; $=(\lambda_f/\lambda_s)(e/x)Pr^m \quad Re_x^n$, where $m=1/3$, $n=1/2$ for laminar boundary layer flow
e	thickness of the flat plate
f	dimensionless stream function; $=\psi/(\nu_f u_{in} x)^{1/2}$, where ψ is the dimensional stream function
L	length of the flat plate used in the numerical calculations
Pr	Prandtl number
q	heat flux
r^*	dimensionless parameter effective in the non-parallel-region; Eq. (8)
Re_x	Reynolds number; $=u_{in}x/\nu_f$
T	temperature
T^*	dimensionless temperature; $=(T-T_{in})/(T_b-T_{in})$
u, v	velocity component
x^*	dimensionless x-coordinate; Eq. (6)
x, y, z	coordinates, see Fig. 1

Greek symbols

ζ	dimensionless z-coordinate; $=z/e$
η	dimensionless y-coordinate; $=y(u_{in}/\nu_f x)^{1/2}$
λ	thermal conductivity
ν_f	kinematic viscosity of the fluid.

Subscripts

b	bottom of the flat plate
f	fluid
in	main stream condition
s	flat plate
w	surface of the flat plate.

$$\begin{cases} \text{at } z=e: \quad T_s = T_b \\ \text{at } x=0: \quad \dfrac{\partial T_s}{\partial x}=0, \quad T_f=T_{in} \\ \text{at } x=\infty: \quad \dfrac{\partial T_s}{\partial x}=0 \\ \text{at } y=z=0: \quad T_f=T_s, \quad -\lambda_f\dfrac{\partial T_f}{\partial y}=\lambda_s\dfrac{\partial T_s}{\partial z} \\ \text{at } y=\infty: \quad T_f=T_{in} \end{cases} \tag{4}$$

where λ_f, λ_s are the thermal conductivities of the fluid and the flat plate, respectively.

3. Dimensionless groups

Since the x-directional conduction in the fluid is neglected, the heat flux in the fluid, q_f, is parallel to the y-axis as shown by a bold arrow in Fig. 1. In the flat plate, both the x- and z-directional conduction are considered. As the effect of x-directional heat conduction is rapidly weakened with x, the flat plate can be divided into two regions: a region near the leading edge of the flat plate where the heat flux in the flat plate, q_s, makes an oblique angle with the z-coordinate (this region will be referred to as the non-parallel-region hereinafter), and a region downward where the heat flux q_s is parallel to the z-coordinate (the parallel-region). q_s's are shown by bold arrows in the flat plate in Fig. 1. The situation can be seen in the isotherm graphs given by Vynnycky et al. (Fig. 2 and Fig. 3 of Ref. [7]).

For the parallel-region, 'the vectorial dimensional analysis which distinguishes phases' described in Ref. [9] gives the following dimensionless relation for the surface temperature of the flat plate

$$T_w^* = \text{function of } (Pr,\ x^*) \tag{5}$$

where T_w^* and x^* are defined as

$$T_w^* = (T_w - T_{in})/(T_b - T_{in}), \quad x^* = \left(\frac{\lambda_s}{\lambda_f}\right)^2 \frac{a_f x}{u_{in} e^2}, \tag{6}$$

and Pr is the Prandtl number of the fluid. Thus, derived dimensionless x-coordinate x^* is substantially equivalent to the so-called Brun number Br_x [1] (actually, for the case of the laminar boundary layer flow a relation $x^{*-1/2} \propto Br_x$ holds).

For the non-parallel-region, where vectorial dimen-

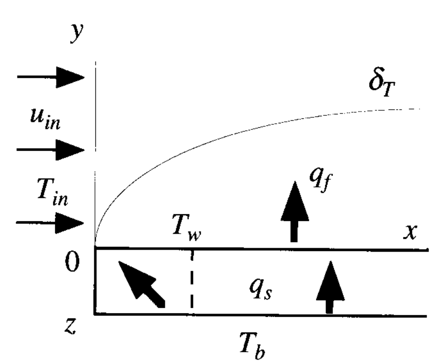

Fig. 1. Schematic diagram of the problem.

sional analysis cannot be applied, Eq. (3) is non-dimensionalized as follows using x^* and $\zeta = z/e$

$$r^{*4}\frac{\partial^2 T_s^*}{\partial x^{*2}} + \frac{\partial^2 T_s^*}{\partial \zeta^2} = 0 \tag{7}$$

where

$$T_s^* = \left(\frac{T_s - T_{in}}{T_b - T_{in}}\right), \quad r^* = \left(\frac{\lambda_s}{\lambda_f}\right)\sqrt{\frac{a_f}{u_{in}e}}. \tag{8}$$

4. Numerical calculation

After the numerical calculations for the flat plate of finite length L, the effect of the finiteness of the plate length is excluded by dropping the calculation results for the rear region of the flat plate (say $x/L > 0.8$). This is reasonable, since the temperature gradient rapidly becomes smaller downstream, so the end effect is limited within a certain distance upstream from the trailing edge of the plate. As the maximum value of y is also limited in numerical calculation, this value, y_{max}, is determined so that the ratio of the x-directional outflow at $x = L$ to the total outflow is about 0.9.

Accordingly, in the third and the last lines in Eq. (4),

at $x = \infty$: at $y = \infty$:

are replace with

at $x = L$: at $y = y_{max}$:

respectively.

The flow field is obtained by numerically integrating Eq. (1) using the bi-section method and the Runge–Kutta method. For reference, the calculated value of $(d^2f/d\eta^2)_{\eta=0}$ and $(v\ Re_x^{1/2}/u_{in})_{\eta=\infty}$ are 0.33206, 0.8605 compared with those of 0.33206, 0.8604 given in Ref. [8].

To obtain the temperature field, Eqs. (2) and (3) are then numerically solved by a FORTRAN program based on the SIMPLE algorithm [10], u, v in Eq. (2) being supplied with the above obtained flow solution. This program, under the constant temperature condition throughout the flat plate, gives values of $Nu_x/Pe_x^{1/2}$ (where Nu_x is the local Nusselt number, Pe_x the local Peclet number) within 7.5% compared with that of the theoretical result $Nu_x/Pe_x^{1/2} = 0.332Pr^{-1/6}$ for the constant wall temperature.

In the present calculations, the flat plate is the epoxy resin, and the fluid is the air [$Pr = 0.72$, $\nu_f = 1.604 \times 10^{-5}$ m^2/s, $\lambda_f = 2.614 \times 10^{-2}$ W/(m K)] or the water [$Pr = 7.1$, $\nu_f = 1.010 \times 10^{-6}$ m^2/s, $\lambda_f = 0.5947$ W/(m K)]. The value of λ_s/λ_f is 11.5 for the air case and 0.504 for the water case. The value of L/e covers from 0.2 to 500.

5. Results and comparisons

The calculated results of the surface temperature of the flat plate for the air and for the water are shown in Fig. 2. Transition from the non-parallel-region to the parallel-region can be seen, and as x^* becomes smaller in the non-parallel-region, the surface temperature approaches some constant value depending on the value of r^*. As the value of r^* approaches zero, the non-parallel-region disappears and the whole flat plate becomes the parallel-region and this is the case treated in Refs. [1–5]. The surface temperature T_w^* for $Pr = 0.01$ by Vynnycky et al. [7] are also shown for reference. Their points are read by a ruler from Fig. 8 of Ref. [7], and rearranged using x^*. The values of r^* are not shown, because their results do not extend to the further smaller x^* region.

In Fig. 3, the surface temperature corresponding to the case $r^* = 0$ (that is, the parallel-region solution) is shown, along with those reported by the previous studies [1,2,5]. Judging from Luikov's [1] developing process, his integral method solution is considered to be valid around $Pr = 1$, and so is Payvar's result [2].

Luikov's solution, upon rearrangement, gives the following equation

$$T_w^* = 1/(1 + 0.331x^{*-1/2}\ Pr^{-1/6}) \tag{9}$$

in the present notation. The line of T_w^* for $Pr = 0.72$ is shown in Fig. 3 irrespective of the validity range of his variable z (here z is defined by $(3/2)(\lambda_f/\lambda_s)(e/\delta_T)$ in Ref. [1], where δ_T is the thickness of the thermal boundary layer). Payvar's points are measured with a

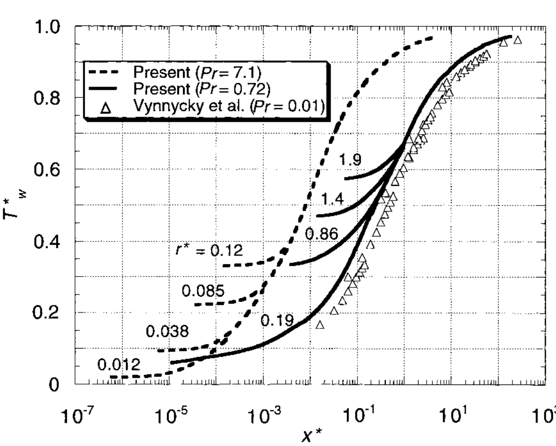

Fig. 2. Surface temperature of the flat plate—present results.

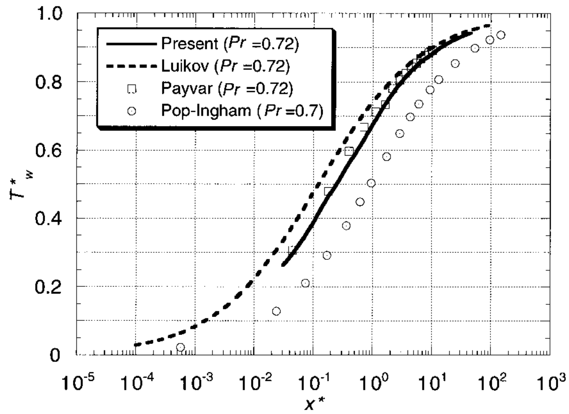

Fig. 3. Parallel-region solution ($r^*=0$)—comparison.

ruler from Fig. 2 of Ref. [2] and the values of T^*_w and x^* are calculated for $Pr = 0.72$ by

$$x^* = 1/(Pr^{1/3}\ Br_x^2), \quad T^*_\mathrm{w} = 1 - \Phi_\mathrm{w} \tag{10}$$

(Br_x, with $m = 1/3$ and $n = 1/2$, and Φ_w follow the definitions of Ref. [2]) on the basis of Eqs. (20), (18), (9), (7) and (17) of Ref. [2]. The present result compares well with those of Luikov [1], Payvar [2] and Karvinen [3]. Although Karvinen's result is not shown here, his result almost coincides with Payvar's. These comprise the first solution family.

The result by Pop and Ingham (Tables 1 and 2 of Ref. [5]) is also shown on the same figure, their dimensionless axial coordinate being converted by

$$x^* = \xi^2/Pr$$

(ξ follows their definition) (11)

on the basis of Eqs. (5a), (5b) and (15b) of Ref. [5]. The result by Pop and Ingham and the result by Pozzi and Lupo (Fig. 2 of Ref. [4]), which agree well with each other, deviate far from the first solution family. This is the second solution family.

If the x^* value of the Pop and Ingham's result were multiplied by $(1/2)^2$, then their result would agree well with the present result, and there would be no two solution families. Any cause of this discrepancy between the two solution families is not known at the present stage.

6. Concluding remarks

The surface temperature of the flat plate under the classical conjugate condition is studied, and it is shown that the flat plate is divided into two regions by the dominant directions of the heat fluxes in the fluid and the flat plate: the non-parallel-region and the parallel-region. r^* is the sole dimensionless parameter in the non-parallel-region and starting from a definite value determined by r^*, the surface temperature first crawls, then it begins to rise and merges into the parallel-region solution. The effective dimensionless x-coordinate throughout the whole region is x^*, which is substantially equivalent to the Brun number ($Br_x \propto x^{*-1/2}$ for the laminar boundary layer flow).

There are two solution families for the parallel-region solution for the surface temperature. Judging from the general validity of the integral method in the laminar boundary layer theory, it is expected that a valid solution should fall into the integral method solution within certain errors around $Pr = 1$. This ensures the validity of the present solution.

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